uGEMM: Unary Computing Architecture for GEMM Applications

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Executive Summary

- Review the demand of energy-efficient GEMM implementations, and motivate uGEMM over other unary approaches.
- Demonstrate uGEMM's compatibility for arbitrarily encoded inputs for multiplication and addition mathematically.
- Prove the knob of uGEMM's high energy efficiency to be early termination enabled by high accuracy and stability.

Outline

- ❑ **Background**
- ❑ Motivation
- ❑ Architecture
- ❑ Evaluation

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GEneral Matrix Multiply (GEMM)

- Ubiquitous in applications
  - Computer vision
  - Signal processing
  - Machine learning
- "At the center of Deep Learning"
  - 95% of GPU runtime
  - 89% of CPU runtime
- Energy efficiency
  - Unary computing as salvation

Y. Jia, "Learning Semantic Image Representations at a Large Scale," PhD thesis.

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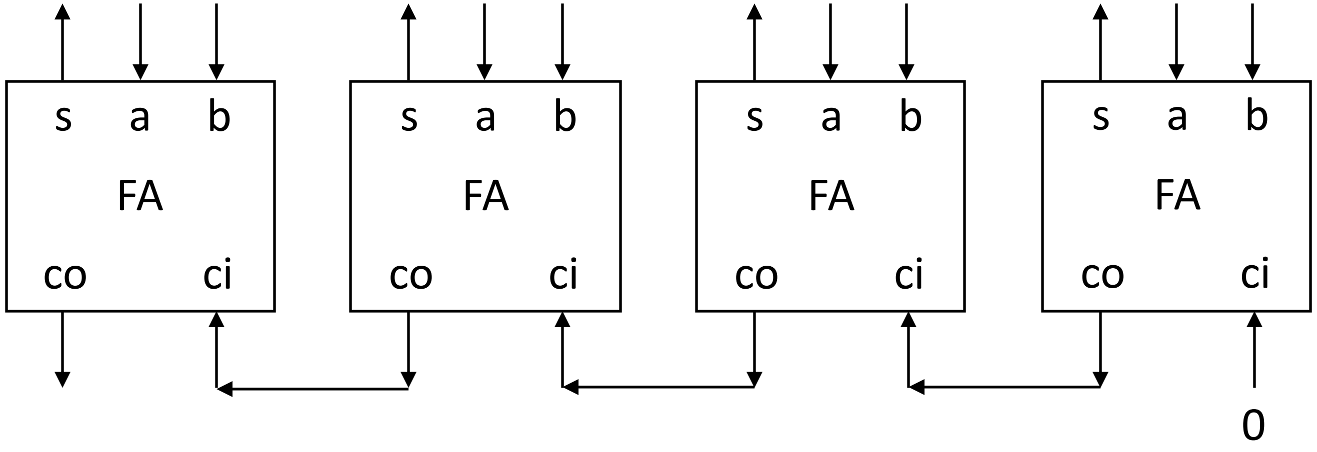
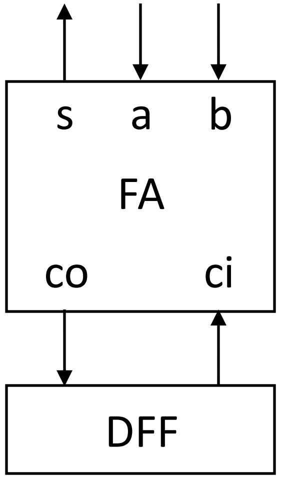
Computing paradigm

Paradigm		Data	Bit significance	Computing domain
Binary computing	Bit parallel	Parallel	Varying	Spatial
	Bit serial			
Unary computing				



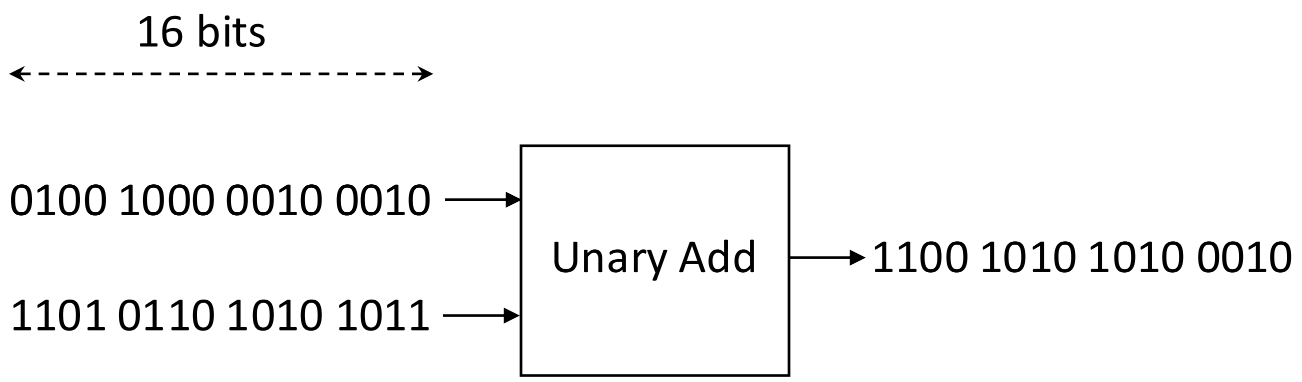
Computing paradigm

Paradigm		Data	Bit significance	Computing domain
Binary computing	Bit parallel	Parallel	Varying	Spatial
	Bit serial	Serial	Varying	Temporal
Unary computing				



Computing paradigm

Paradigm		Data	Bit significance	Computing domain
Binary computing	Bit parallel	Parallel	Varying	Spatial
	Bit serial	Serial	Varying	Temporal
Unary computing		Serial	Equal	Temporal

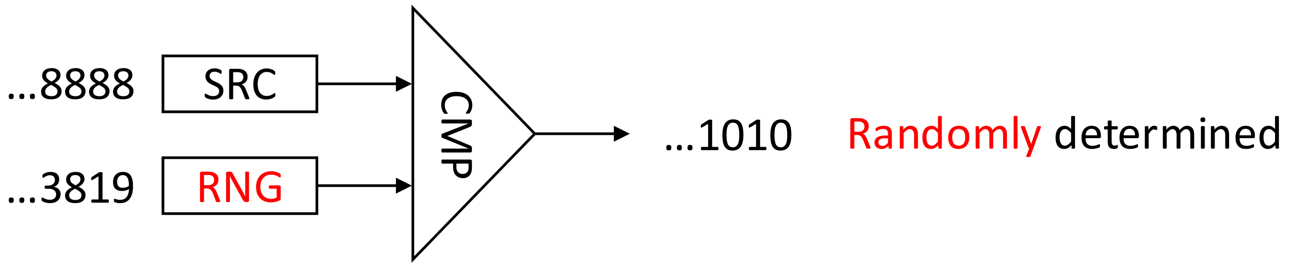


Unary computing scheme

Scheme	Bit stream	Application
Stochastic computing	Rate coding	LDPC, image processing, machine learning
Race logic		

Bit stream as data.
Probability of 1s matters only.

1101 1000 1010 1010 8 1s

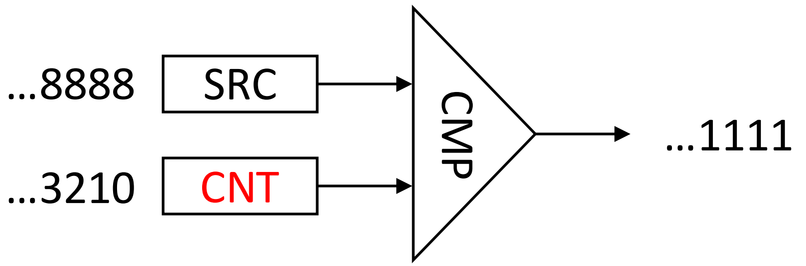


Unary computing scheme

Scheme	Bit stream	Application
Stochastic computing	Rate coding	LDPC, image processing, machine learning
Race logic	Temporal coding	DNA sequencing, decision tree, sorting

Bit stream as data.
Delay of 1s matters only.

0000 0000 1111 1111 8 1s



Deterministically determined

Unary computing data – bit stream

Polarity	Binary width	Unary length	Probability of 1s	Value	Range
Unipolar	N	2^N	P(1)	P(1)	Unsigned, [0, 1]
Bipolar					

1101 1000 1010 1010 8 1s Unipolar value=0.5

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Unary computing data – bit stream

Polarity	Binary width	Unary length	Probability of 1s	Value	Range
Unipolar	N	2^N	P(1)	P(1)	Unsigned, [0, 1]
Bipolar				2*P(1)-1	Signed, [-1, 1]

1101 1000 1010 1010 8 1s Bipolar value=0.0

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Measure of bit streams

Metric	Bit stream	Goal
Correlation	Two	How similar two bit streams are.
Stability		

+1 correlation: count of aligned 1s is maximized.

1101 1000 1010 1010   8 1s
1101 1000 1010 1010   8 1s

Aligned 1 count: 8

Expected in temporal coding

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Measure of bit streams

Metric	Bit stream	Goal
Correlation	Two	How similar two bit streams are.
Stability		

+1 correlation: count of aligned 1s is maximized.

1101 1000 1010 1010	8 1s	Aligned 1 count: 8
1101 1000 1010 1010	8 1s	

0 correlation: count of aligned 1s is balanced.

1101 1000 1010 1010	8 1s	Aligned 1 count: 4
1110 0100 0101 1010	8 1s	

Expected in rate coding

Measure of bit streams

Metric	Bit stream	Goal
Correlation	Two	How similar two bit streams are.
Stability		

+1 correlation: count of aligned 1s is maximized.
- 1101 1000 1010 1010 — 8 1s
- 1101 1000 1010 1010 — 8 1s

Aligned 1 count: 8

0 correlation: count of aligned 1s is balanced.
- 1101 1000 1010 1010 — 8 1s
- 1110 0100 0101 1010 — 8 1s

Aligned 1 count: 4

-1 correlation: count of aligned 1s is minimized.
- 1101 1000 1010 1010 — 8 1s
- 0010 0111 0101 0101 — 8 1s

Aligned 1 count: 0

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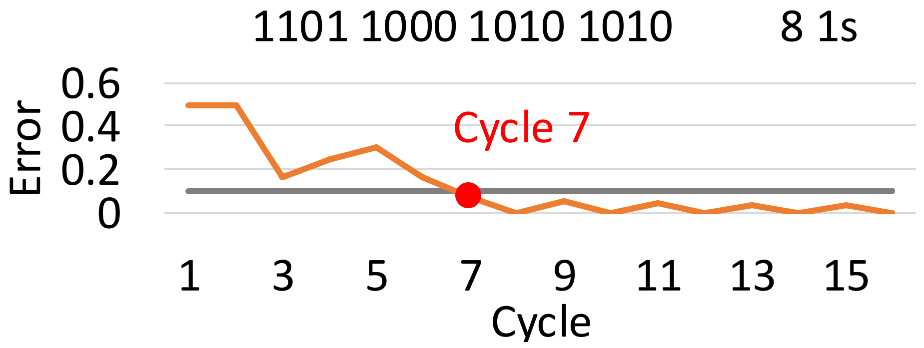
Measure of bit streams

Metric	Bit stream	Goal
Correlation	Two	How similar two bit streams are.
Stability	One	How fast a bit stream converges to its desired value.



Stability = 1-(7-1)/16 = 0.625

Stable point:

Error never exceeds a given threshold from now on.

Stability:

Ratio of stable cycle count to all.

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Measure of bit streams

Metric	Bit stream	Goal
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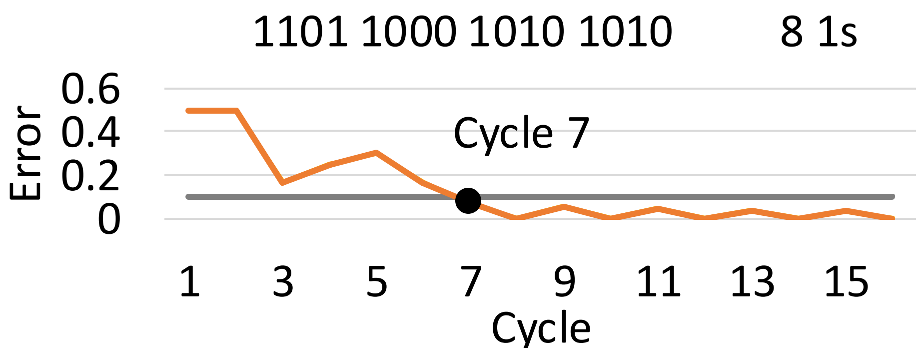
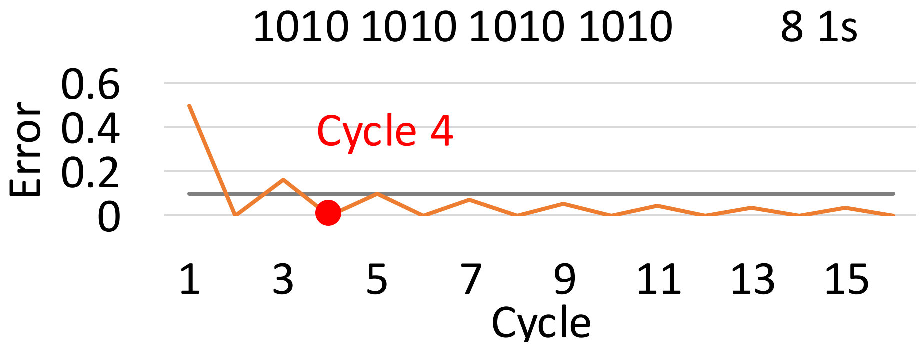


Stability = 1-(7-1)/16 = 0.625



Stability = 1-(4-1)/16 = 0.8125

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Measure of bit streams

Metric	Bit stream	Goal
Correlation	Two	How similar two bit streams are.
Stability	One	How fast a bit stream converges to its desired value.

Stable point:
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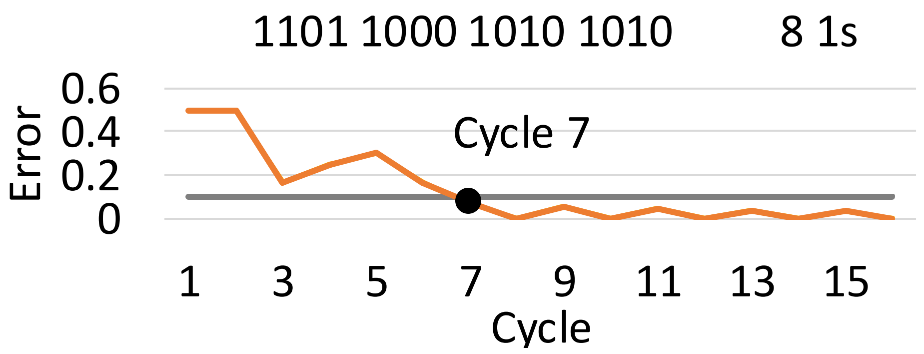
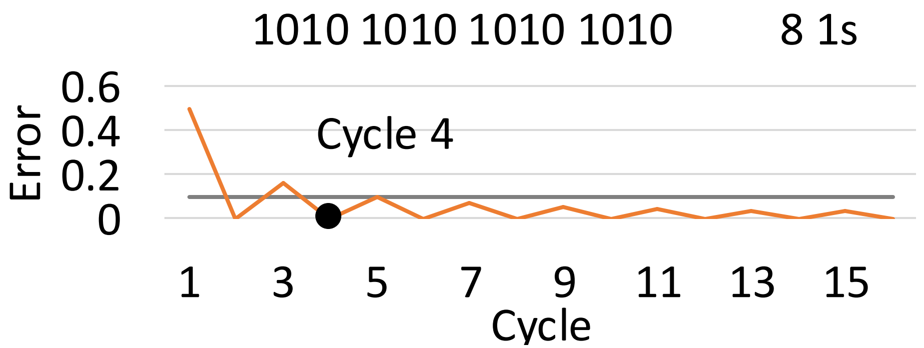
Stability:
Ratio of stable cycle count to all.



Stability = 1-(7-1)/16 = 0.625

High stability enables early termination.



Stability = 1-(4-1)/16 = 0.8125

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Outline

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- ❑ **Motivation**
- ❑ Architecture
- ❑ Evaluation

Unified uGEMM

- Unified unary General Matrix Multiplication
  - First to support Add/Mul in temporal coding

Function	Rate coding	Temporal coding
Add	[14, 16, 19, 25], uGEMM	uGEMM
Multiply	[14, 16, 19, 57], uGEMM	uGEMM
Add constant		[39, 63]
Minimum	[4, 32]	[38, 63]
Maximum	[4, 32]	[38, 63]
Inhibit		[59, 63]

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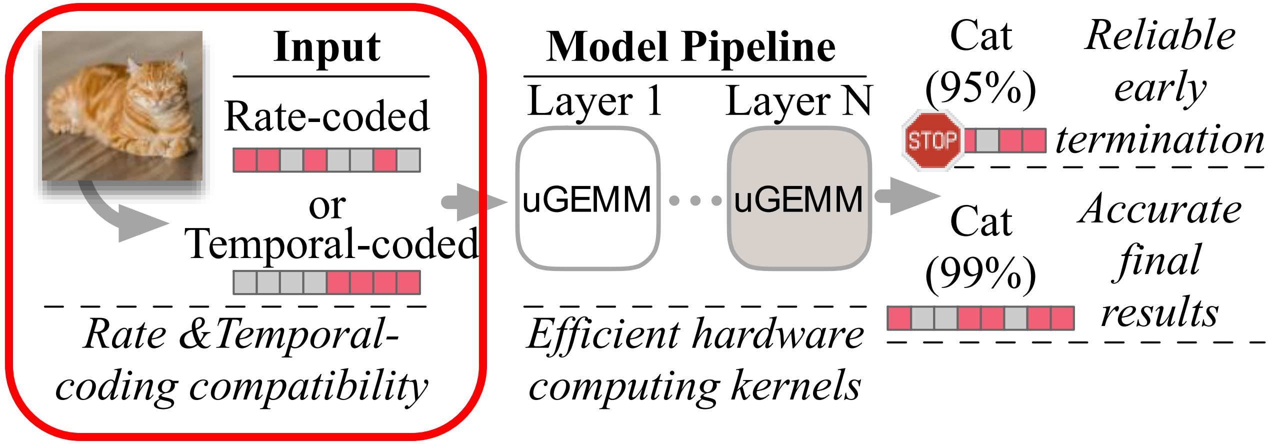
Unified uGEMM

- Unified unary General Matrix Multiplication
  - First to support Add/Mul in temporal coding
  - Compatibility for varying coding and polarity



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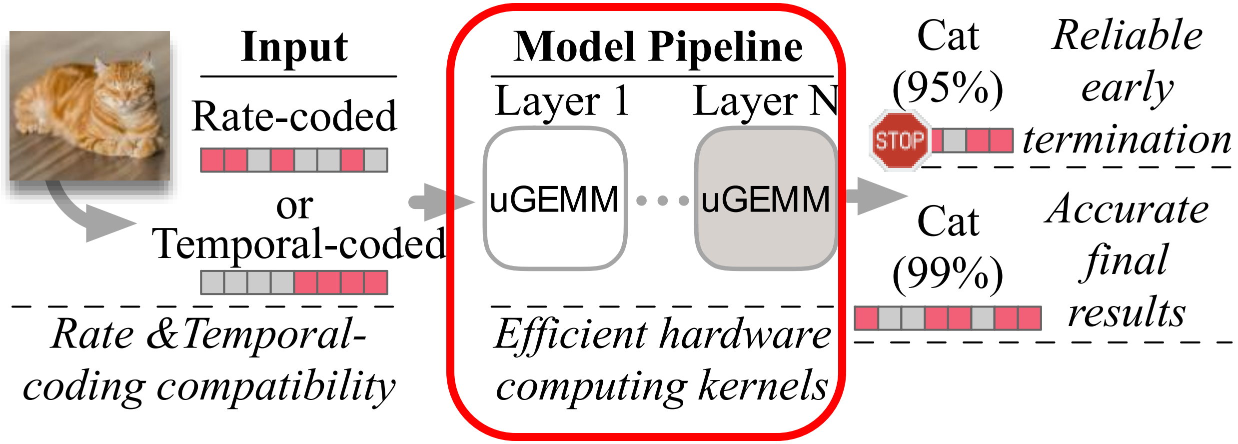
Unified uGEMM

- Unified unary General Matrix Multiplication
  - First to support Add/Mul in temporal coding
  - Compatibility for varying coding and polarity
  - Early termination
    - Fully streaming computation



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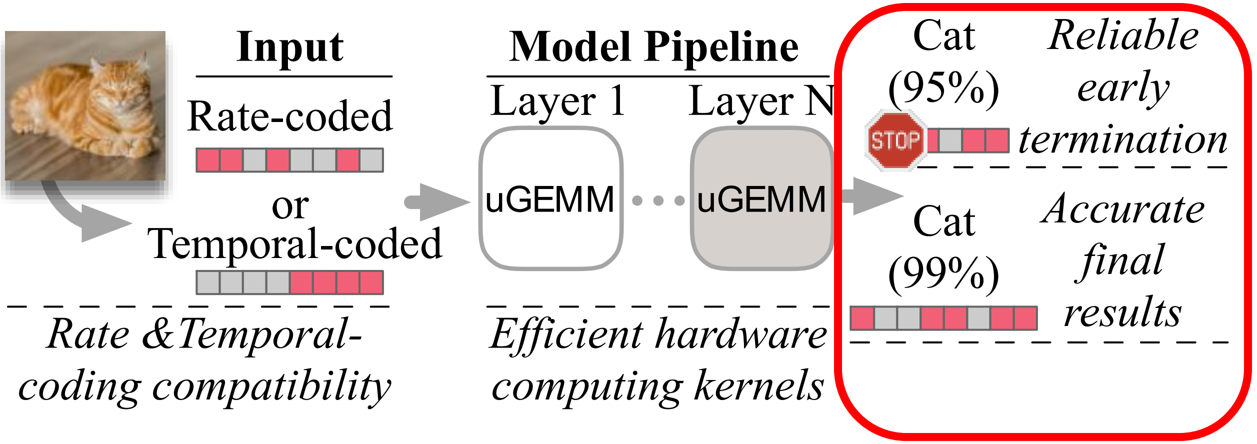
Unified uGEMM

- Unified unary General Matrix Multiplication
  - First to support Add/Mul in temporal coding
  - Compatibility for varying coding and polarity
  - Early termination
    - Fully streaming computation
    - High accuracy and stability by solving the correlation problem



Outline

- ❑ Background
- ❑ Motivation
- ❑ **Architecture**
- ❑ Evaluation

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uGEMM overview

- Multiplication
  - uMUL
    - Unipolar and Bipolar

- Addition
  - Scaled (uSADD)
    - Unipolar and Bipolar
  - Non-Scaled (uNSADD)
    - Unipolar and Bipolar (first support)

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uGEMM – Multiplication

- uMUL: unipolar
  - Expected function

$$V_{out} = V_{in,0} \cdot V_{in,1},$$

$$P(S_{out} = 1) = P(S_{in,0} = 1) \cdot P(S_{in,1} = 1),$$

uGEMM – Multiplication

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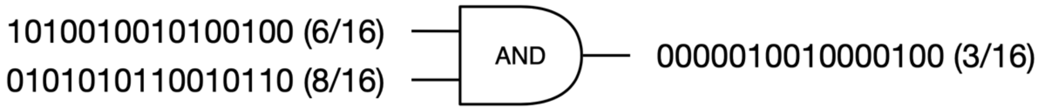
$$V_{out} = V_{in,0} \cdot V_{in,1},$$
$$P(S_{out} = 1) = P(S_{in,0} = 1) \cdot \boxed{P(S_{in,1} = 1)},$$



  - Actual AND gate function

$$P(S_{out} = 1) = P(S_{in,0} = 1, S_{in,1} = 1)$$
$$= P(S_{in,0} = 1) \cdot \boxed{P(S_{in,1} = 1 | S_{in,0} = 1)}$$

uGEMM – Multiplication

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  - Expected function

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$$P(S_{out} = 1) = P(S_{in,0} = 1) \cdot P(S_{in,1} = 1),$$

3/16 6/16 8/16=0.5

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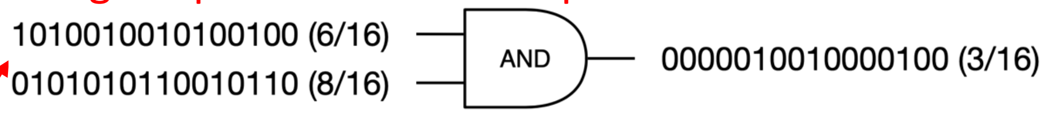
$$P(S_{out} = 1) = P(S_{in,0} = 1, S_{in,1} = 1)$$

$$= P(S_{in,0} = 1) \cdot P(S_{in,1} = 1 | S_{in,0} = 1)$$

3/6 = 0.5

Correct with 0 correlation:

Marginal prob = conditional prob



uGEMM – Multiplication

- uMUL: unipolar
  - Expected function

$$V_{out} = V_{in,0} \cdot V_{in,1},$$

$$P(S_{out} = 1) = P(S_{in,0} = 1) \cdot P(S_{in,1} = 1),$$

3/16 6/16 8/16=0.5

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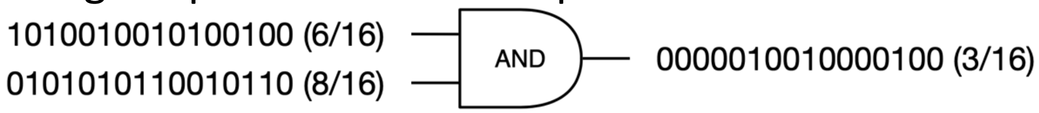
$$P(S_{out} = 1) = P(S_{in,0} = 1, S_{in,1} = 1)$$
$$= P(S_{in,0} = 1) \cdot P(S_{in,1} = 1 | S_{in,0} = 1)$$

6/6 = 1.0

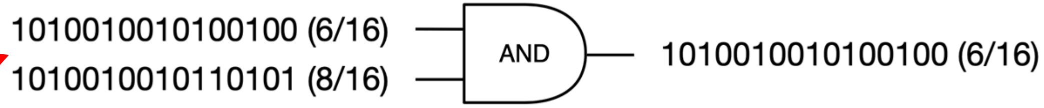
Correct with 0 correlation:

Marginal prob = conditional prob



Wrong with non-0 correlation:

Marginal prob != conditional prob



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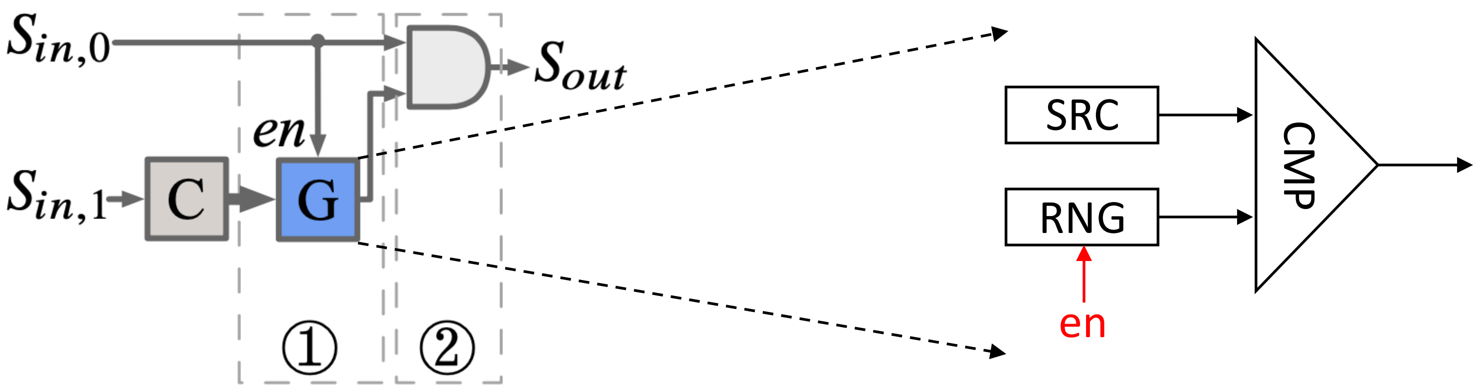
uGEMM – Multiplication

- uMUL: unipolar
  - Conditional bit stream generation
    - Enforce $P(S_{in,1}=1) = P(S_{in,1}=1 | S_{in,0}=1)$



C: Counter
G: Bit stream generator

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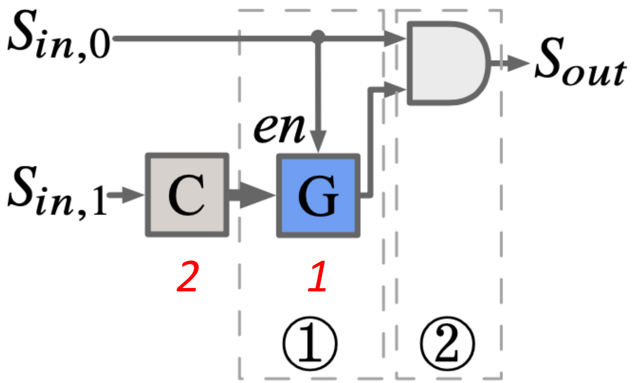
uGEMM – Multiplication

- uMUL: unipolar

2/4: 1 0 0 1

$S_{in,1}$ is static in counter. 2/4



0 0 0 1

2/4*2/4=1/4

Cycle	Counter (C)	RNG (G)	AND 0 (In 0)	AND 1	Output
1	2 >	1	1	1	1
2					
3					
4					

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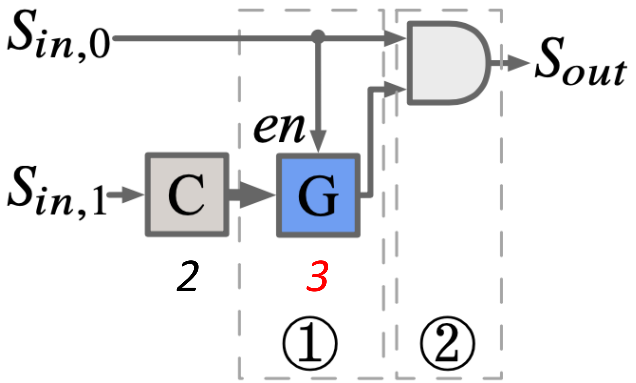
uGEMM – Multiplication

➢ uMUL: unipolar

2/4: 1 0 0 1



0 0 0 1

$S_{in,1}$ is static in counter. 2/4

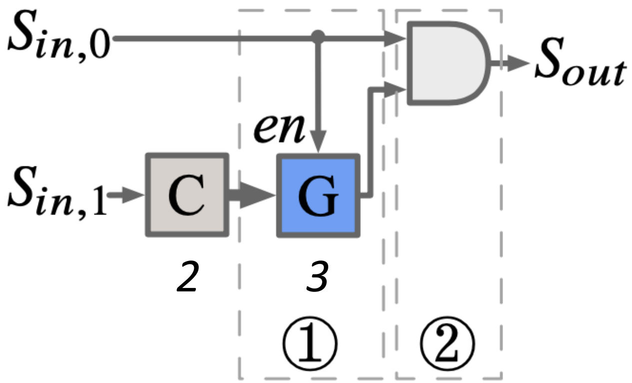
2/4*2/4=1/4

Cycle	Counter (C)	RNG (G)	AND 0 (In 0)	AND 1	Output
1	2	1 en	1	1	1
2	2 <	3	0	0	0
3					
4					

uGEMM – Multiplication

- uMUL: unipolar

2/4: *1 0 0 1*



0 0 0 1

$S_{in,1}$ is static in counter. 2/4

2/4*2/4=1/4

Cycle	Counter (C)	RNG (G)	AND 0 (In 0)	AND 1	Output
1	2	1	1	1	1
2	2	3	0	0	0
3	2 <	3	0	0	0
4					

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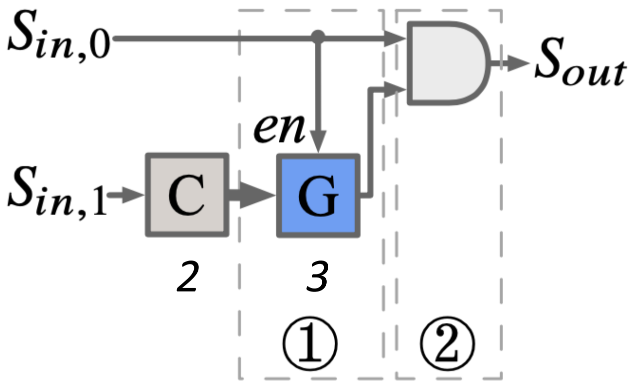
uGEMM – Multiplication

- uMUL: unipolar

2/4: 1 0 0 1

$S_{in,1}$ is static in counter. 2/4



0 0 0 1

2/4*2/4=1/4

Cycle	Counter (C)	RNG (G)	AND 0 (In 0)	AND 1	Output
1	2	1	1	1	1
2	2	3	0	0	0
3	2	3	0	0	0
4	2 <	3	1	0	0

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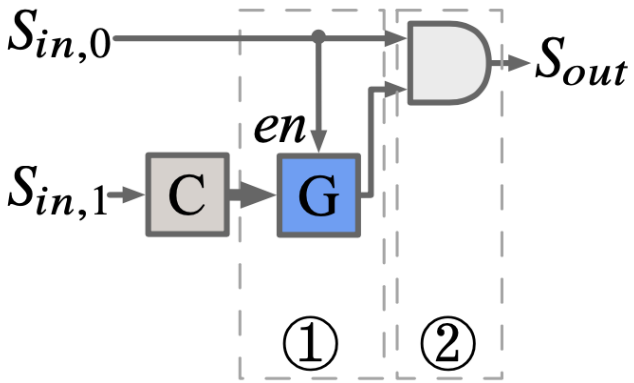
uGEMM – Multiplication

- uMUL: unipolar

2/4: *1 0 0 1*

$S_{in,1}$ is static in counter. 2/4



0 0 0 1

2/4*2/4=1/4

Cycle	Counter (C)	RNG (G)	AND 0 (In 0)	AND 1	Output
1	2	1	1	1	1
2	2	3	0	0	0
3	2	3	0	0	0
4	2	3	1	0	0

1/4

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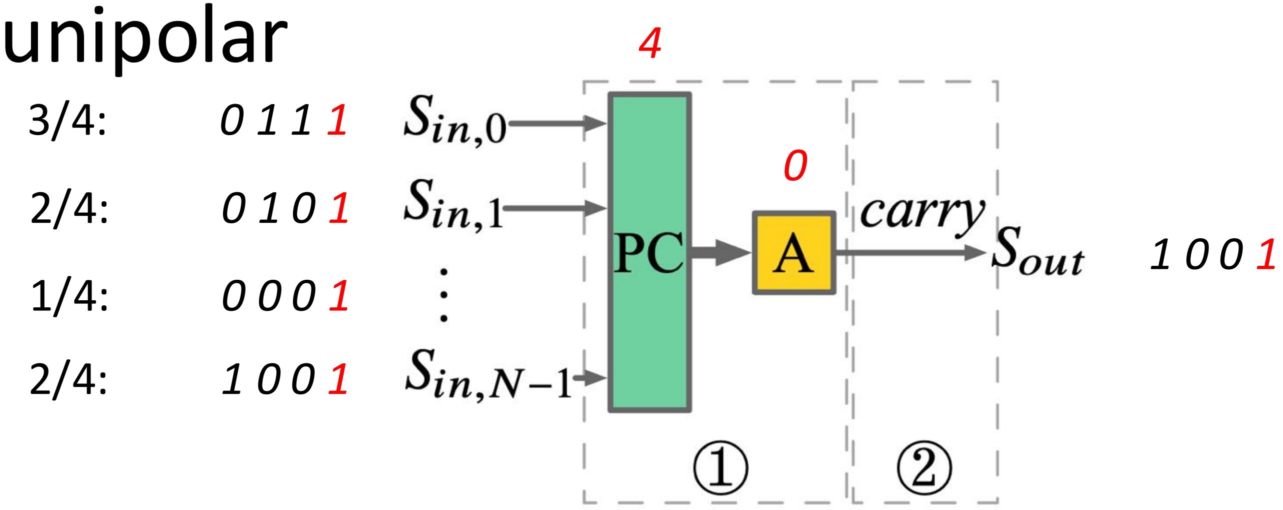
uGEMM – Scaled addition

- uSADD: unipolar

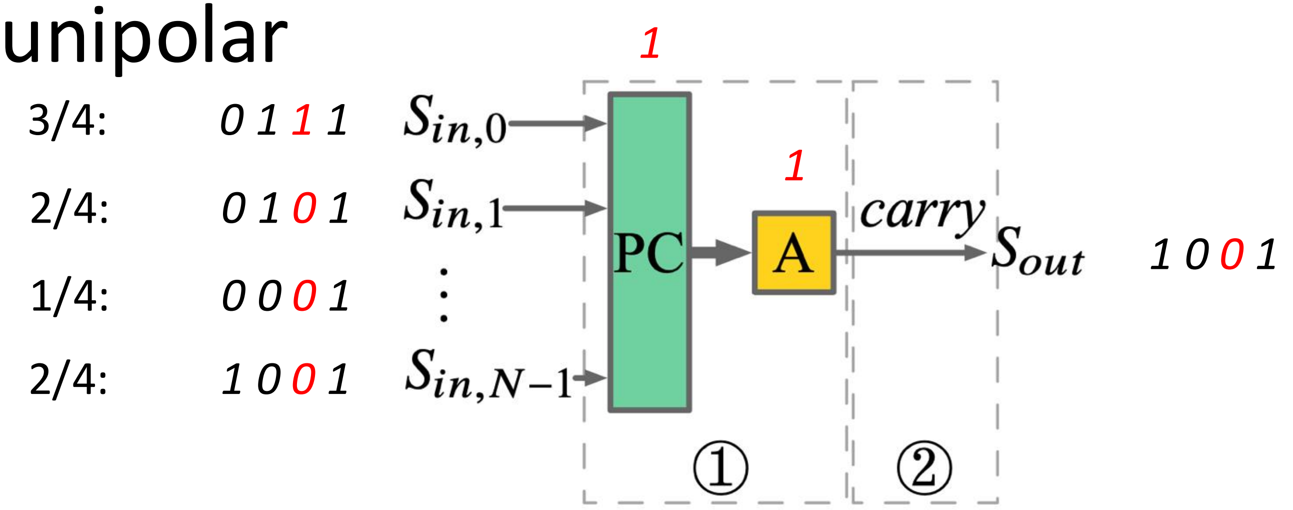


(3/4+2/4+1/4+2/4)/4=2/4

Cycle	Input count (PC)	Accumulator (A)	Carry (output)
1	4	0	1
2			
3			
4			

uGEMM – Scaled addition

- uSADD: unipolar



(3/4+2/4+1/4+2/4)/4=2/4

Cycle	Input count (PC)	Accumulator (A)	Carry (output)
1	4	0	1
2	1	1	0
3			
4			

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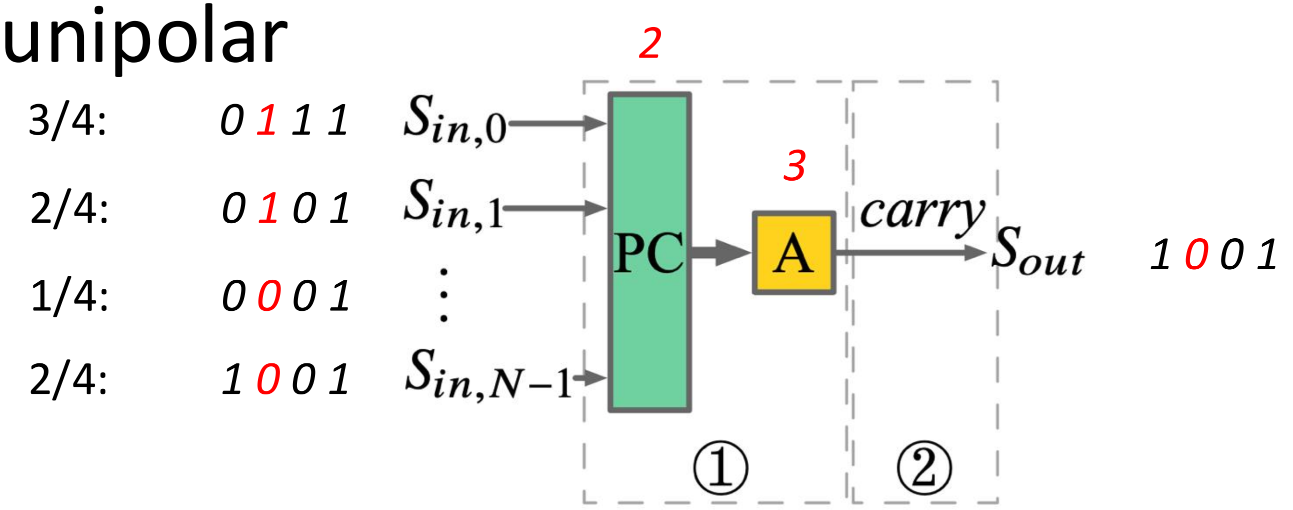
uGEMM – Scaled addition

- uSADD: unipolar



(3/4+2/4+1/4+2/4)/4=2/4

Cycle	Input count (PC)	Accumulator (A)	Carry (output)
1	4	0	1
2	1	1	0
3	2	3	0
4			

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uGEMM – Scaled addition

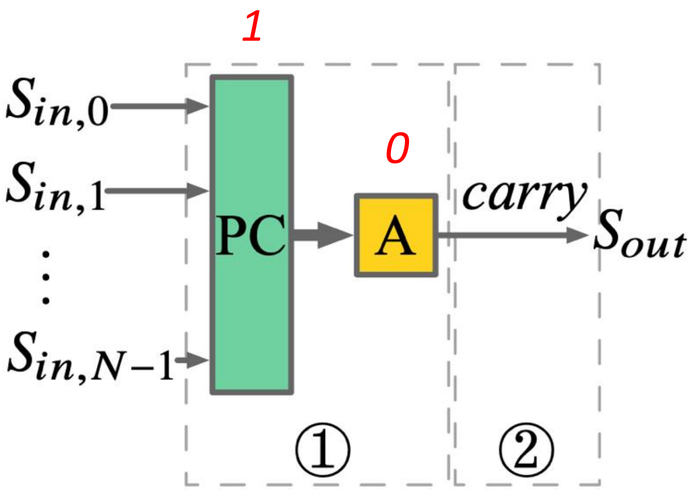
- uSADD: unipolar

3/4: 0 1 1 1

2/4: 0 1 0 1

1/4: 0 0 0 1

2/4: 1 0 0 1



1 0 0 1

(3/4+2/4+1/4+2/4)/4=2/4

Cycle	Input count (PC)	Accumulator (A)	Carry (output)
1	4	0	1
2	1	1	0
3	2	3	0
4	1	0	1

uGEMM – Scaled addition

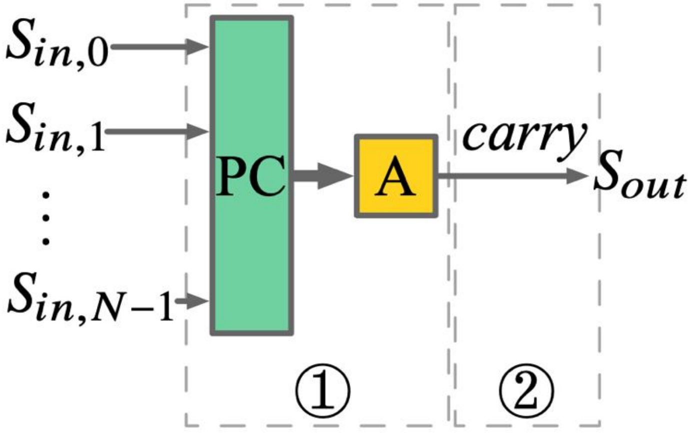
- uSADD: unipolar

3/4: *0 1 1 1*

2/4: *0 1 0 1*

1/4: *0 0 0 1*

2/4: *1 0 0 1*



1 0 0 1

(3/4+2/4+1/4+2/4)/4=2/4

Cycle	Input count (PC)	Accumulator (A)	Carry (output)
1	4	0	1
2	1	1	0
3	2	3	0
4	1	0	1

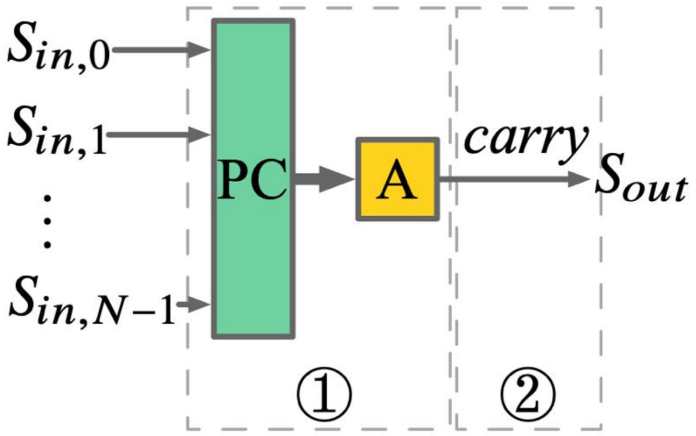
2/4

uGEMM – Scaled addition

- uSADD
  - Expected function

$$V_{out} = \frac{1}{N} \cdot \sum_{n=0}^{N-1} V_{in,n}$$

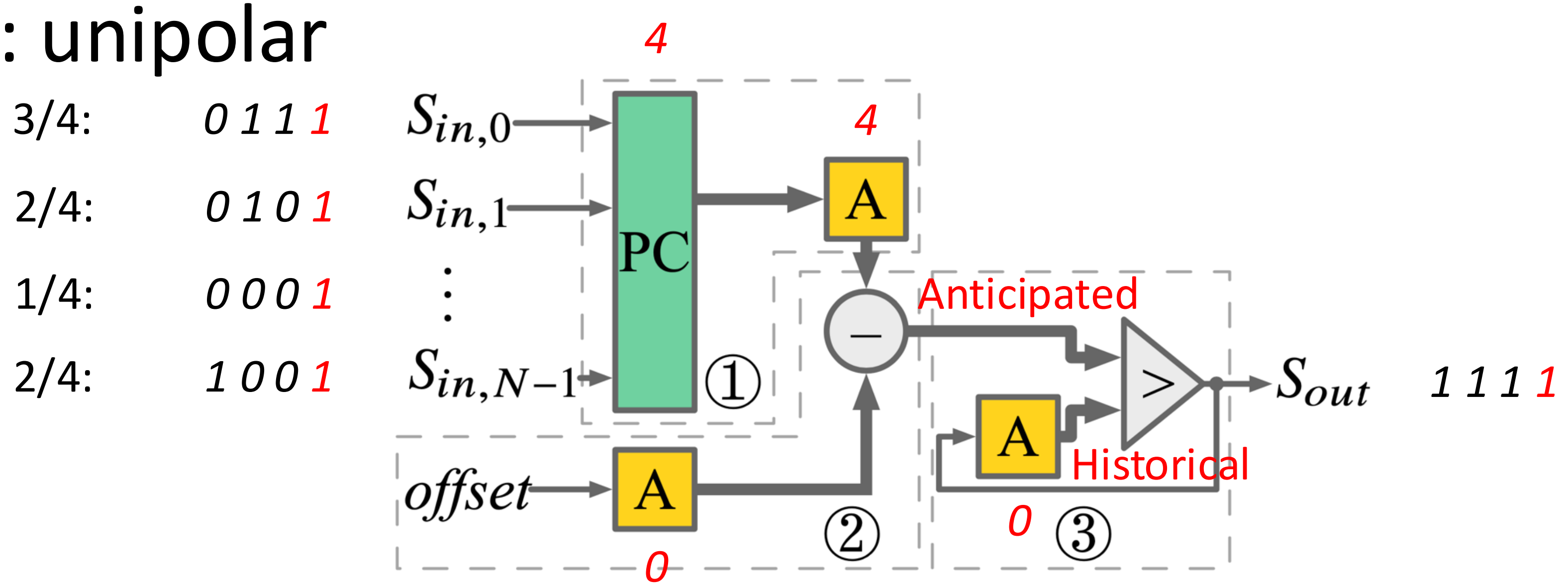
$$\mathbb{C}_{out} = \frac{1}{N} \cdot \sum_{n=0}^{N-1} \mathbb{C}_{in,n}$$



  - Carry overflow mechanism
    - Parallel counter (PC) records current input.
    - When accumulation (A) overflows, output logic 1.

uGEMM – Non-scaled addition

- uNSADD: unipolar



Clip(3/4+2/4+1/4+2/4)=1

Cycle	Input count (PC)	Acc (1)	Acc (3)	Acc (1>3)	Output
1	4	4	0	True →	1
2					
3					
4					

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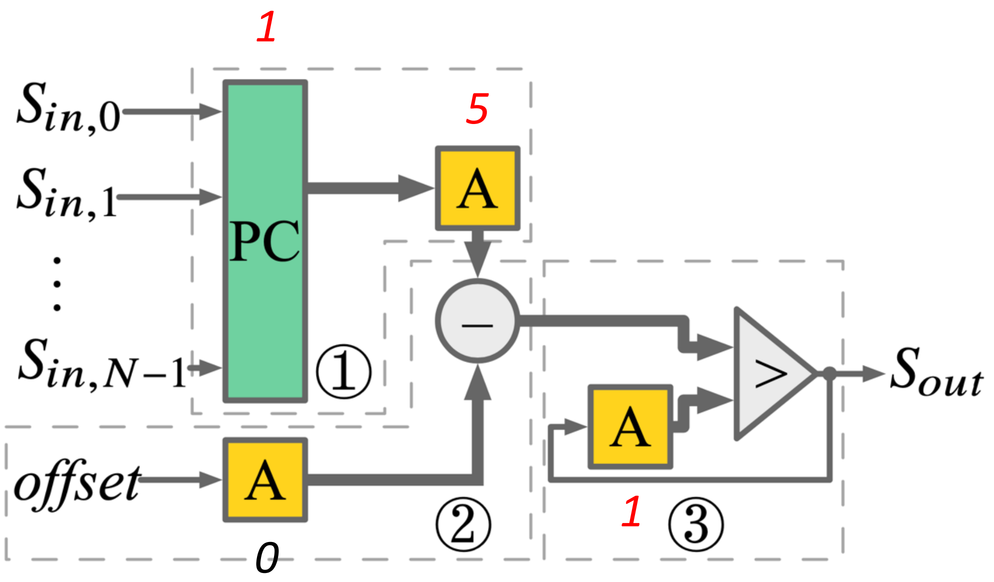
uGEMM – Non-scaled addition

- uNSADD: unipolar

3/4:	0 1 1 1
2/4:	0 1 0 1
1/4:	0 0 0 1
2/4:	1 0 0 1



Output: 1 1 1 1

Clip(3/4+2/4+1/4+2/4)=1

Cycle	Input count (PC)	Acc (1)	Acc (3)	Acc (1>3)	Output
1	4	4	0	True	1
2	1	5	1	True	1
3					
4					

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uGEMM – Non-scaled addition

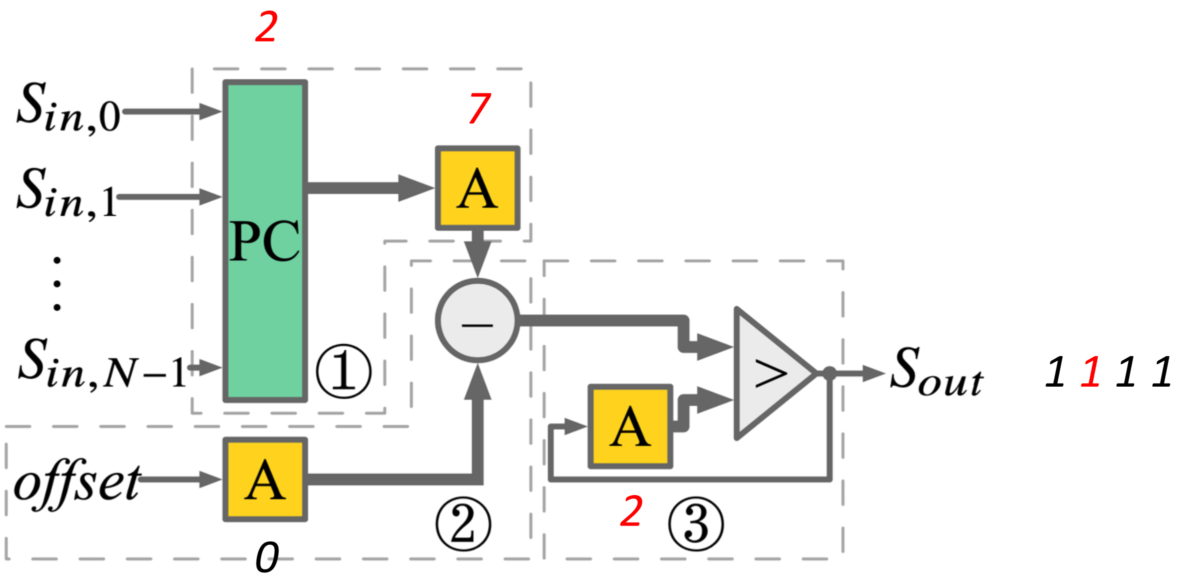
- uNSADD: unipolar

3/4: 0 1 1 1

2/4: 0 1 0 1

1/4: 0 0 0 1

2/4: 1 0 0 1



Clip(3/4+2/4+1/4+2/4)=1

Cycle	Input count (PC)	Acc (1)	Acc (3)	Acc (1>3)	Output
1	4	4	0	True	1
2	1	5	1	True	1
3	2	7	2	True	1
4					

uGEMM – Non-scaled addition

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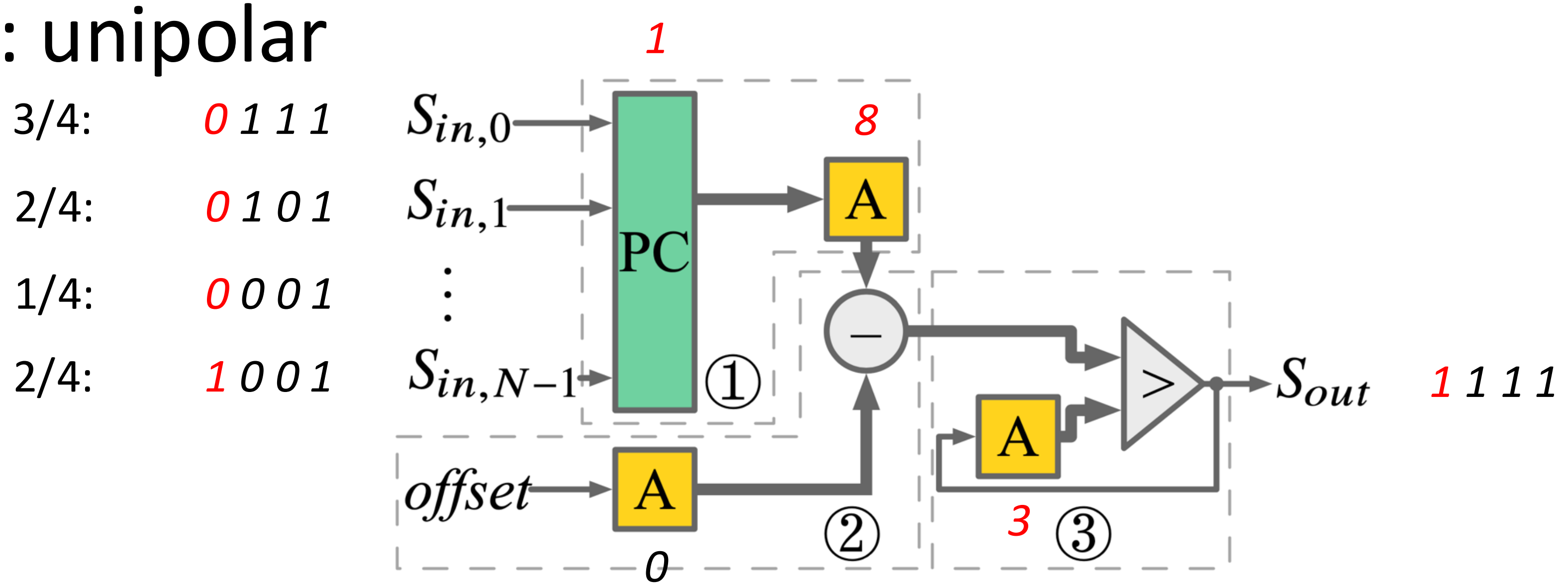

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3	2	7	2	True	1
4	1	8	3	True	1

uGEMM – Non-scaled addition

- uNSADD: unipolar

3/4: *0 1 1 1*

2/4: *0 1 0 1*

1/4: *0 0 0 1*

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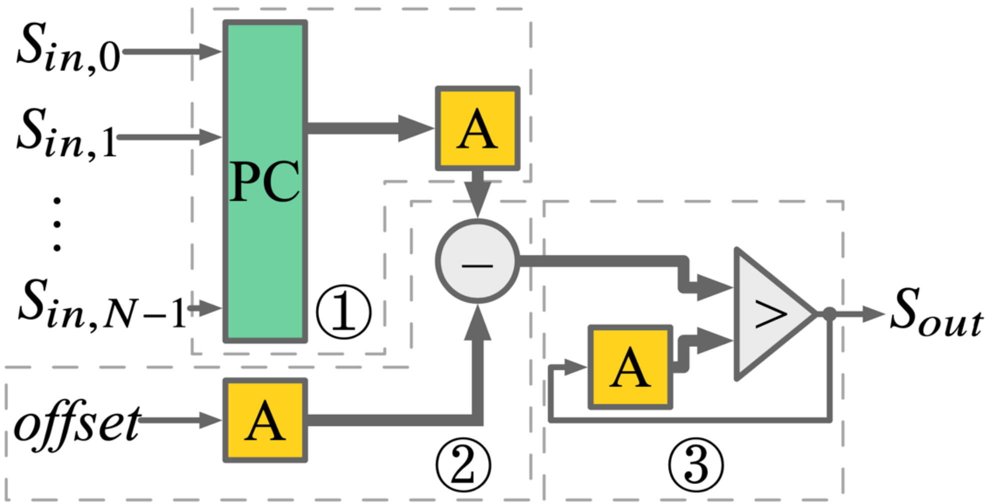

1 1 1 1

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Cycle	Input count (PC)	Acc (1)	Acc (3)	Acc (1>3)	Output
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4	1	8	3	True	1

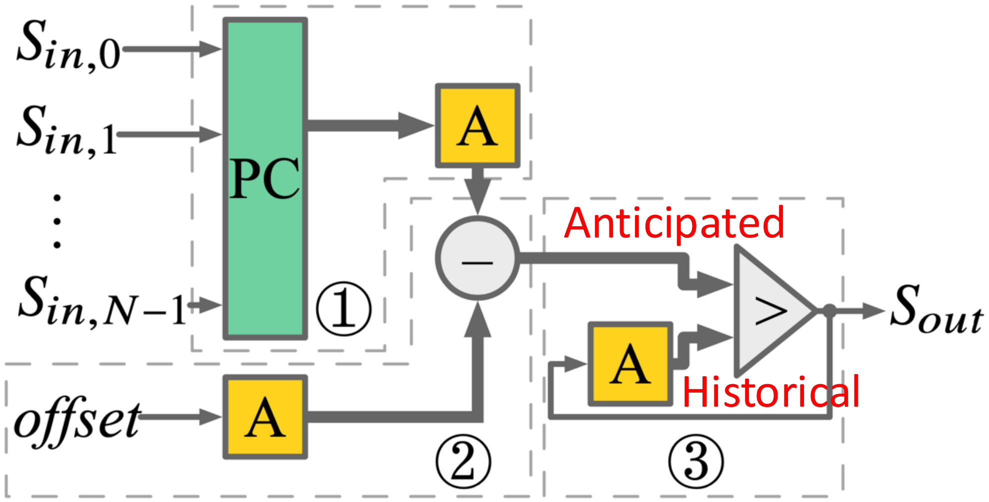
4/4

uGEMM – Non-scaled addition

- uNSADD
  - Expected function

$$V_{out} = \mathrm{clip}\left(\sum_{n=0}^{N-1} V_{in,n}, -k, 1\right)$$

$$\mathbb{C}_{out} = \min\left(\sum_{n=0}^{N-1} \mathbb{C}_{in,n} - L \cdot \mathbb{C}_{offset}, L\right)$$



  - Output tracking mechanism
    - Difference between anticipated and historical output 1s indicates what to output.

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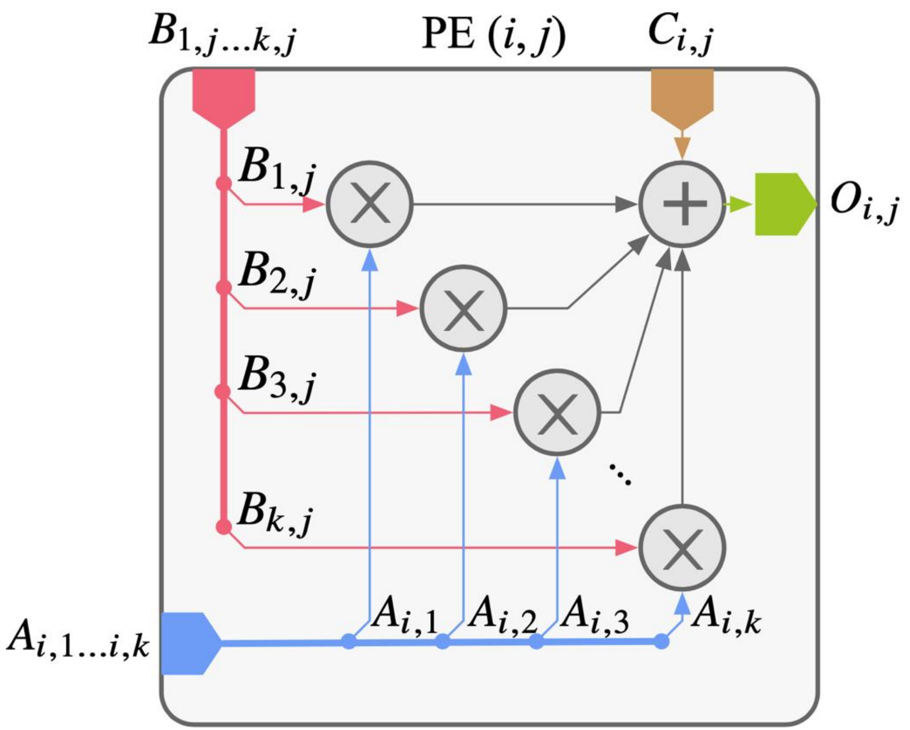
uGEMM – Processing element

- Input insensitivity
  - Multiplication
    - Generating operand conditionally
    - Ignoring correlation



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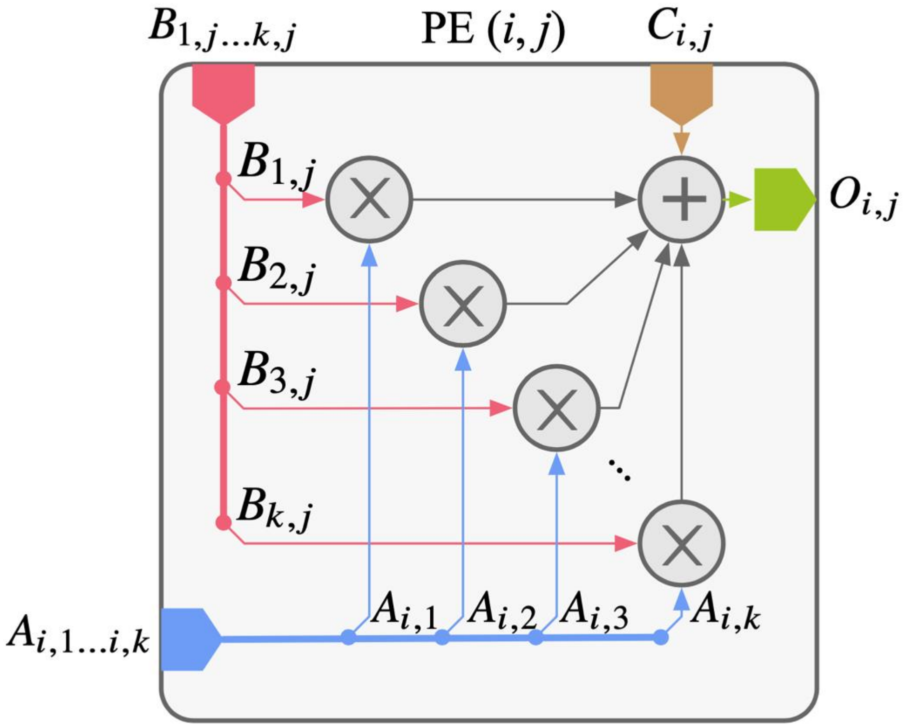
uGEMM – Processing element

- Input insensitivity
  - Multiplication
    - Generating operand conditionally
    - Ignoring correlation
  - Addition
    - Caring about bit count
    - Ignoring bit distribution



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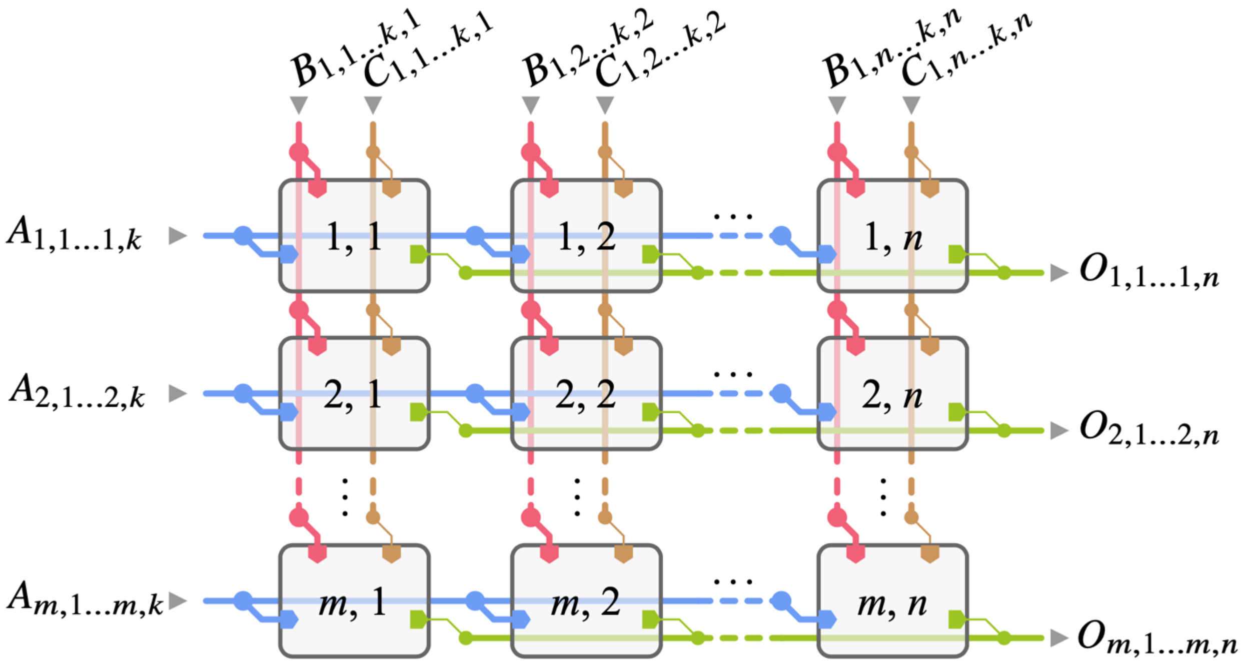
uGEMM – PE array

- High parallelism via broadcasting
- Input insensitivity due to PE
- Reliable early termination
  - Fully streaming computation
  - High accuracy and stability



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Outline

- ❑ Background
- ❑ Motivation
- ❑ Architecture
- ❑ **Evaluation**

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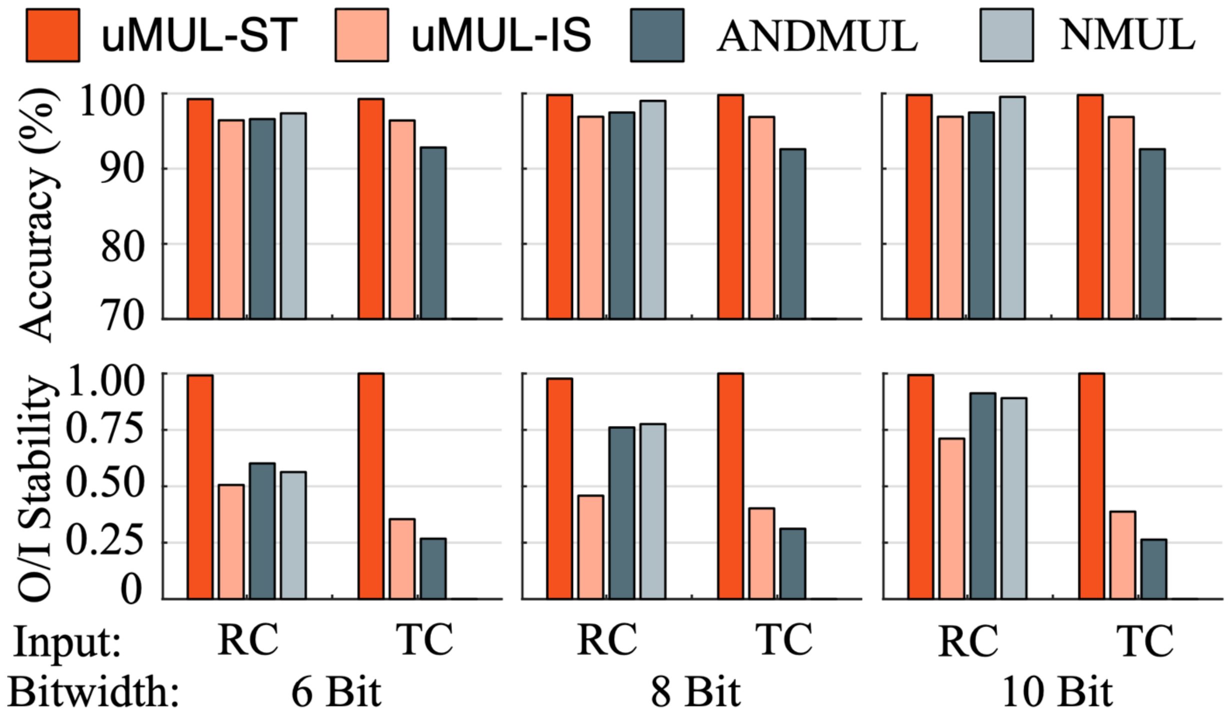
uMUL performance: Unipolar

- ➢ Accurate final result
  - Static input (ST)
  - In-stream input (IS)
- ➢ Faster stabilization
  - Output/Input stability
- ➢ Input insensitivity
  - Rate coding (RC)
  - Temporal coding (TC)



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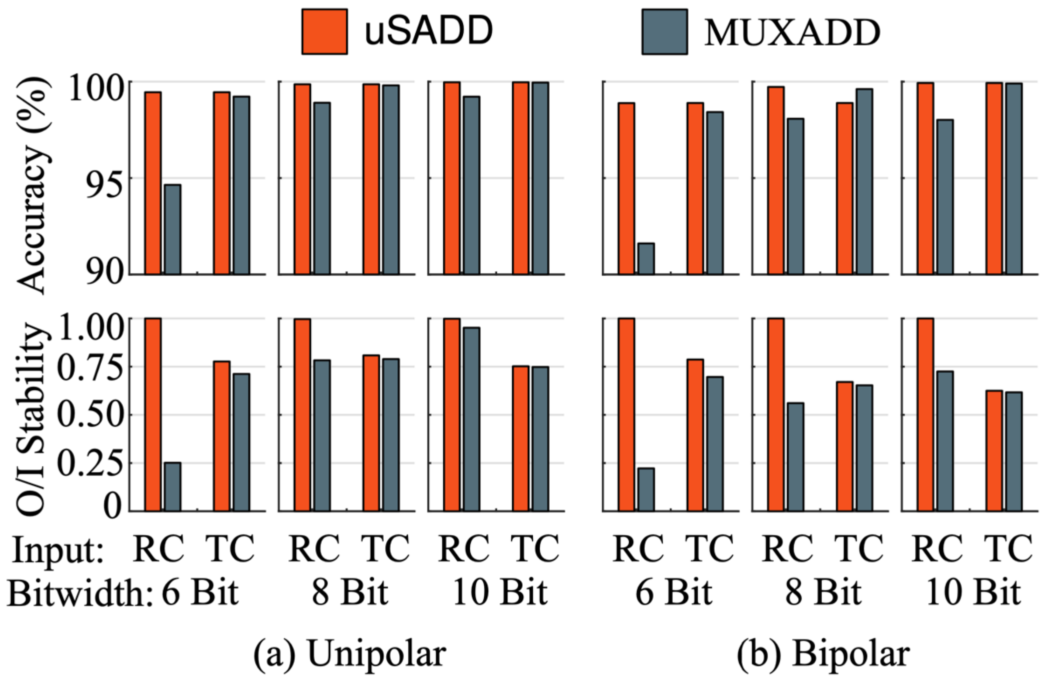
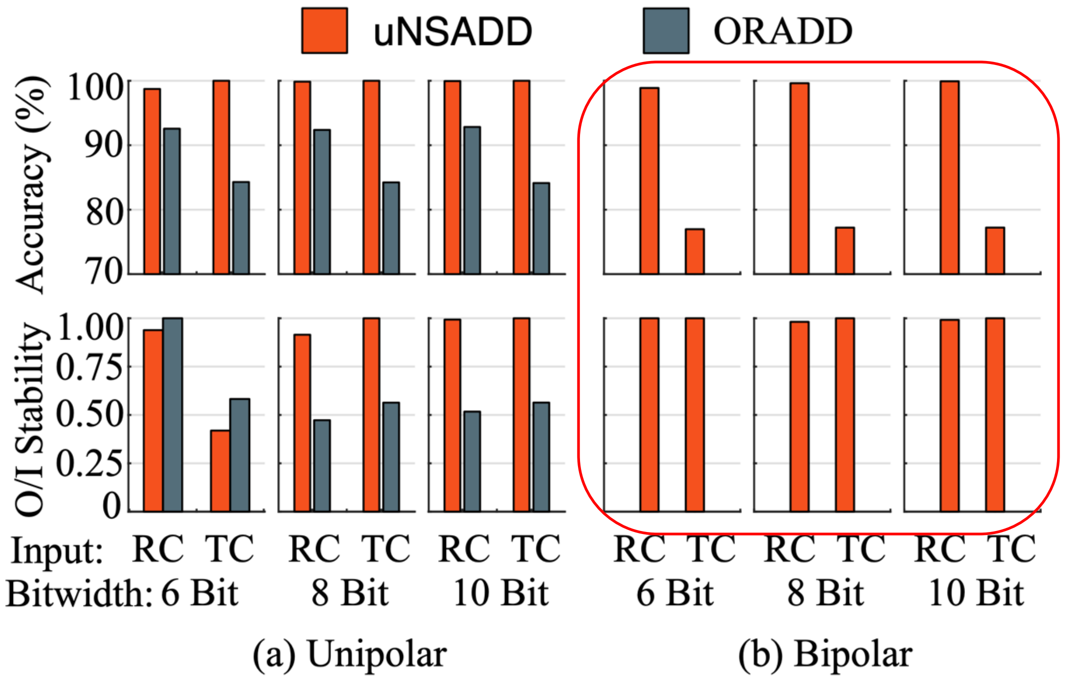
ADD performance: uSADD and uNSADD

- ➢ Accurate final result
- ➢ Faster stabilization
- ➢ Input insensitivity

First to support bipolar non-scaled addition





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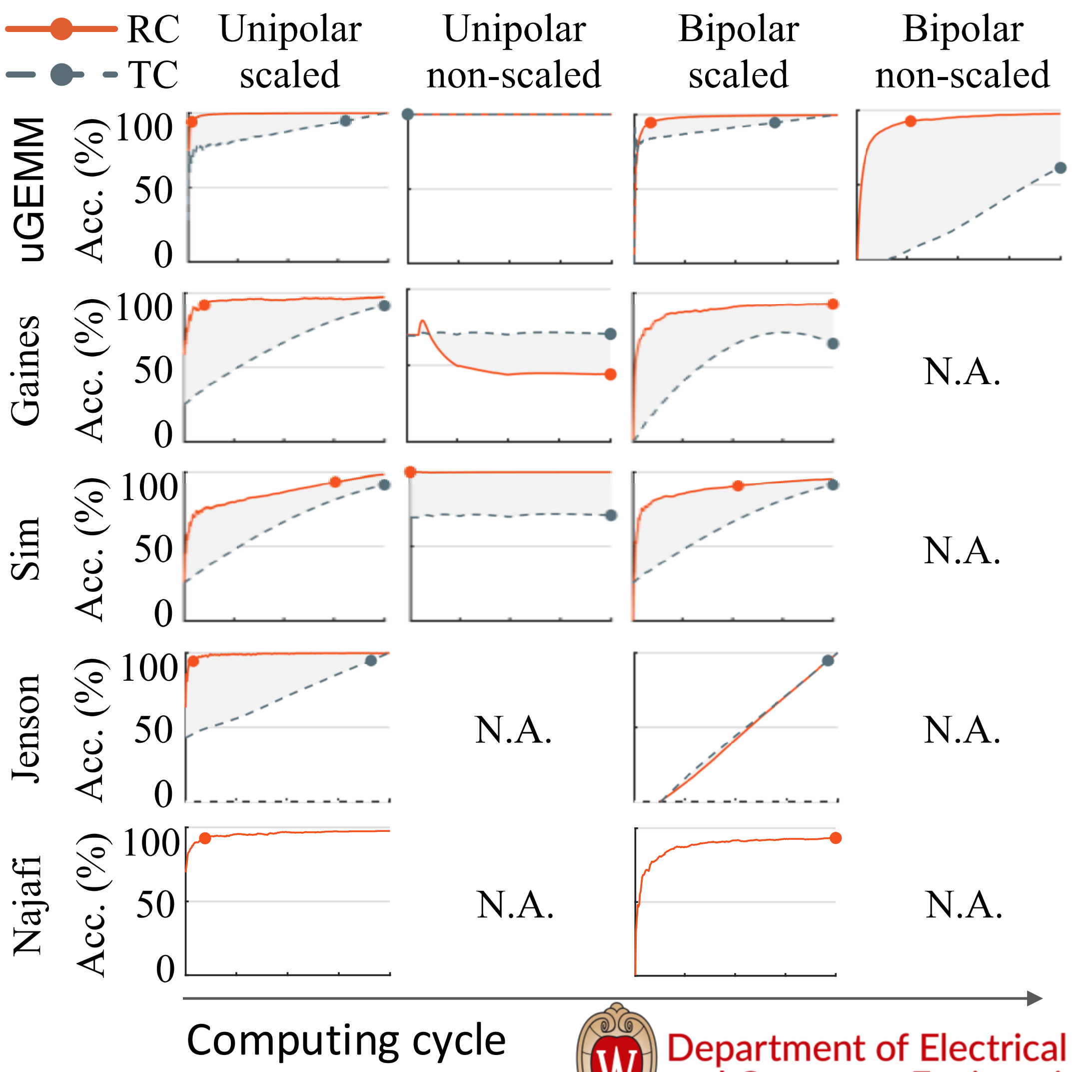
uGEMM performance

- Accurate final result
  - Cross out all configurations



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uGEMM performance

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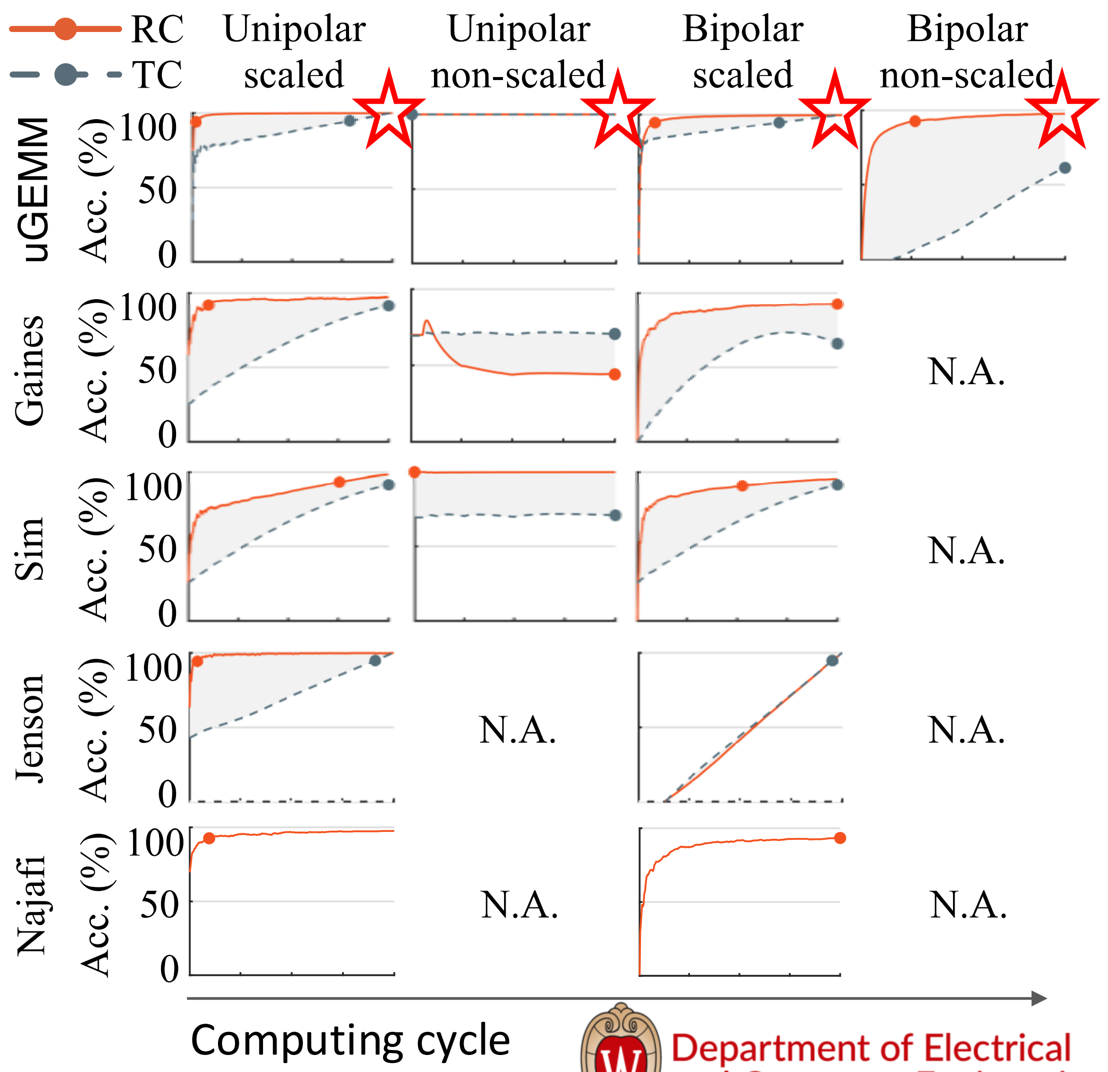

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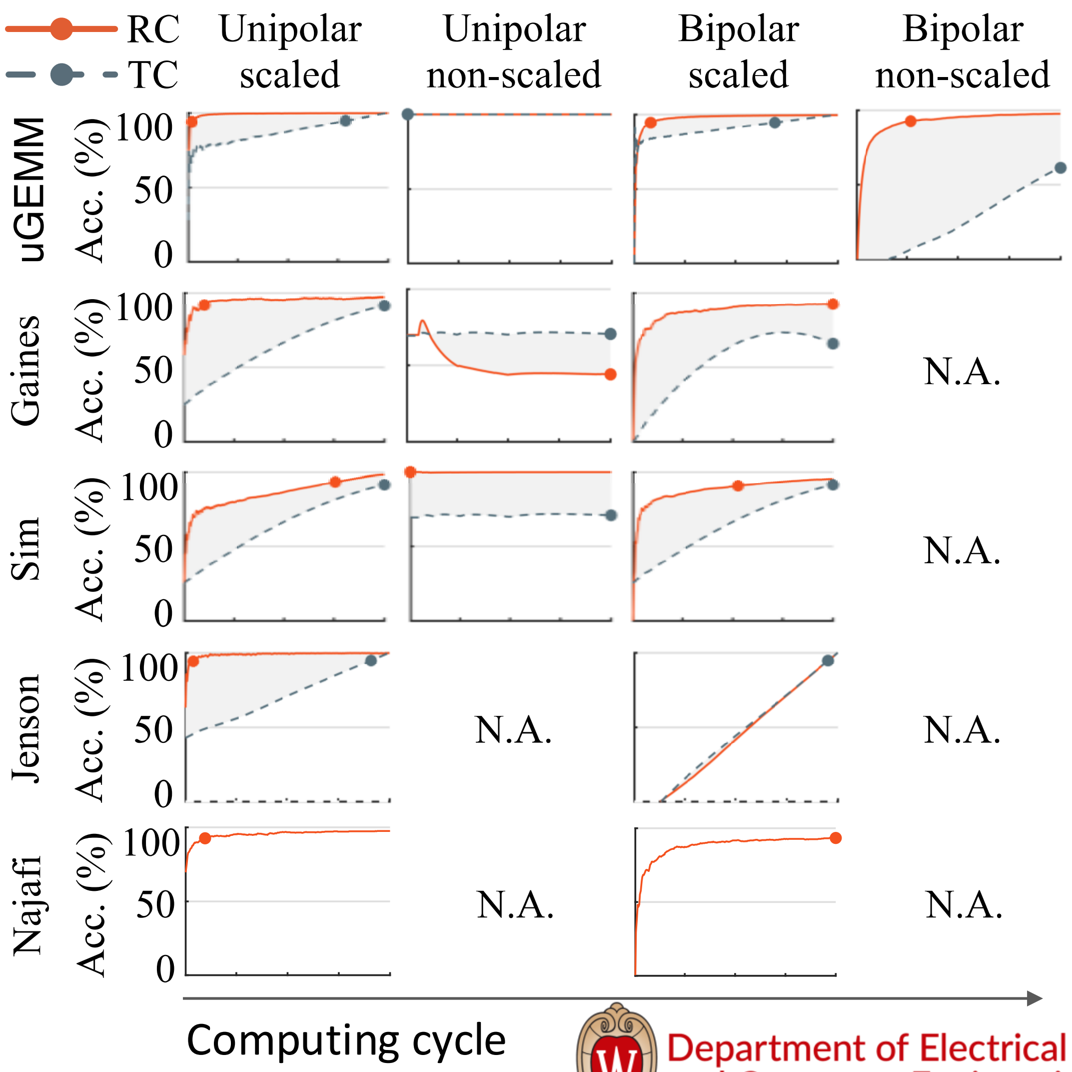
uGEMM performance

- Accurate final result
    - Cross out all configurations
- Reliable early termination
    - Earlier stable point



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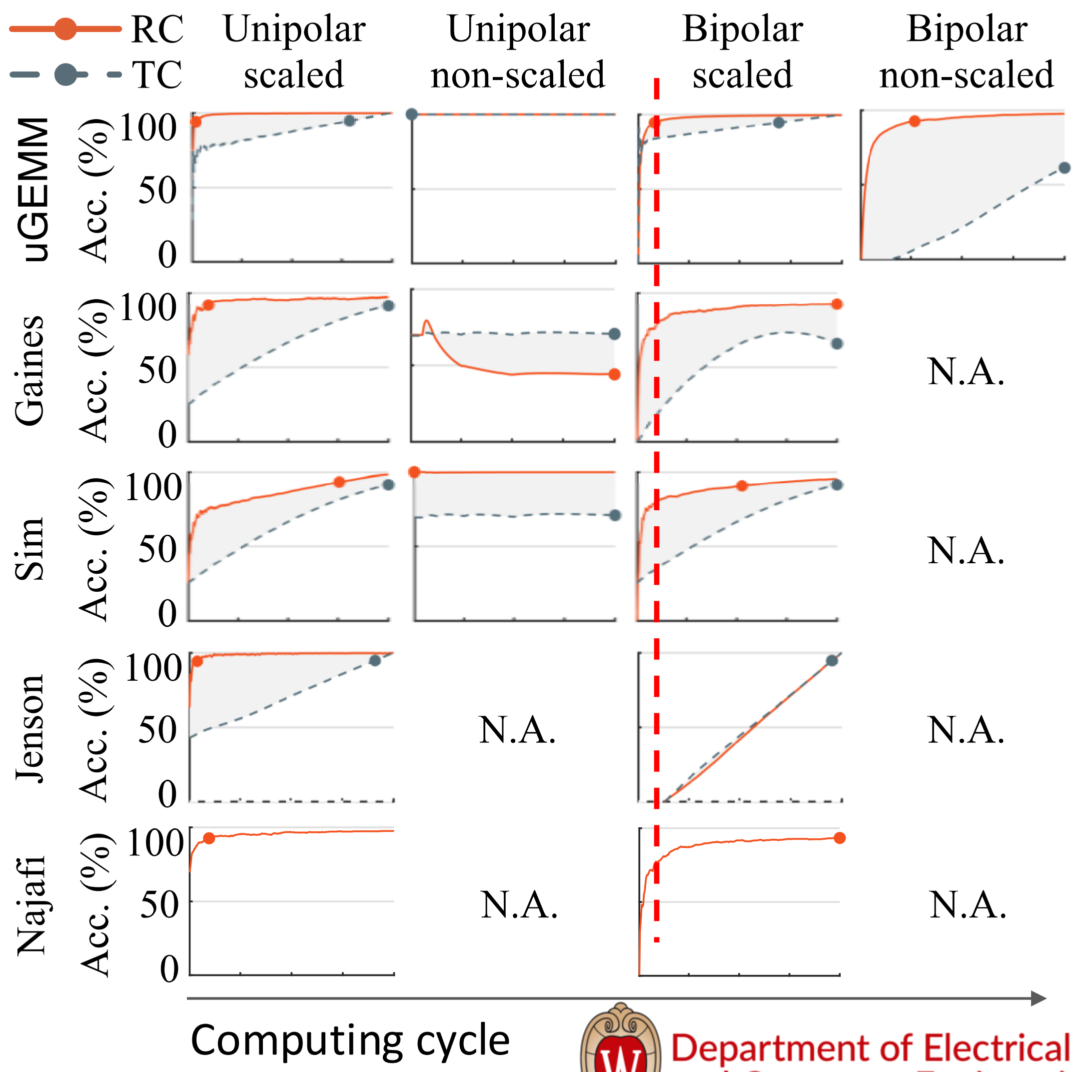

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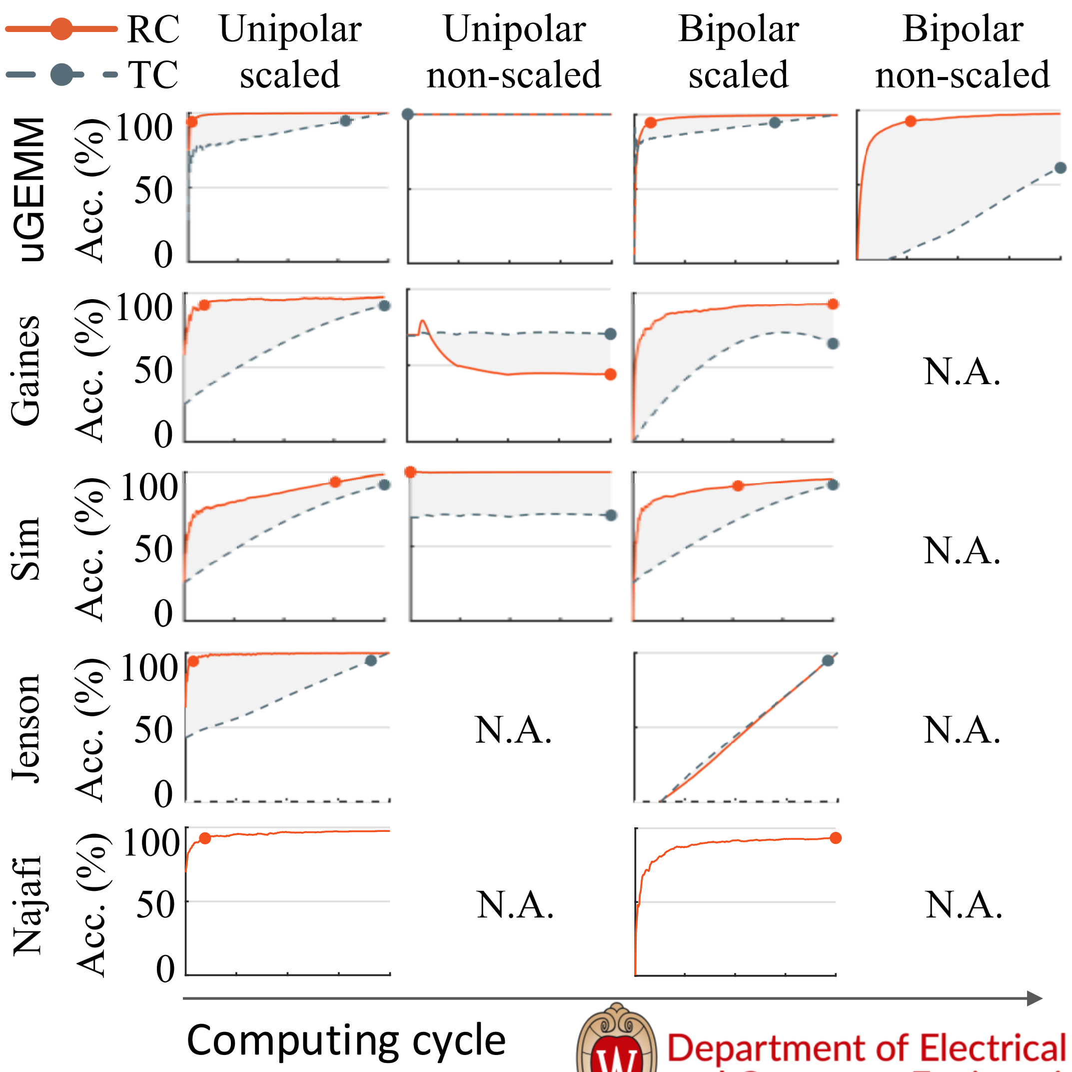
uGEMM performance

- Accurate final result
  - Cross out all configurations
- Reliable early termination
  - Earlier stable point
- Input insensitivity
  - Minimal RC/TC difference



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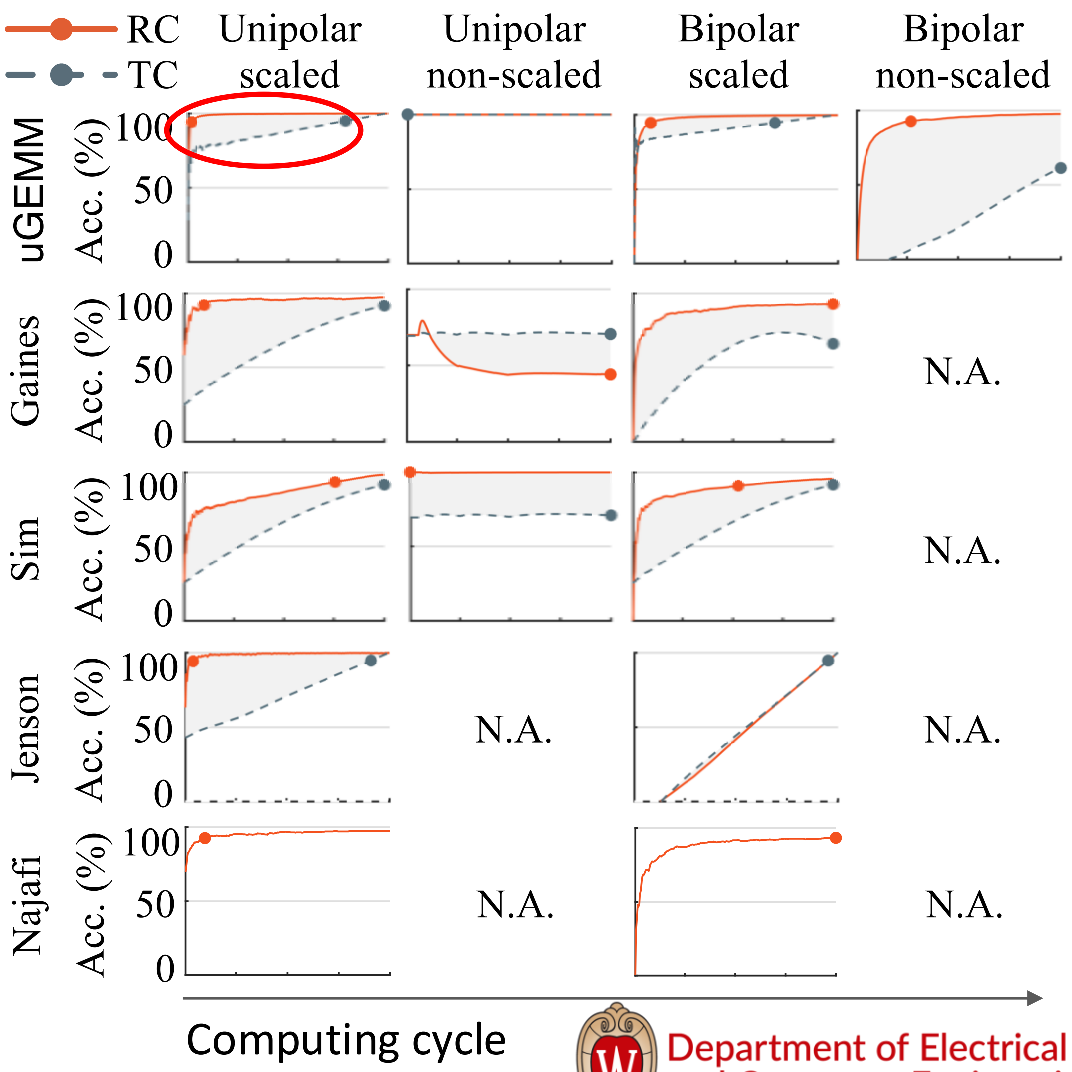

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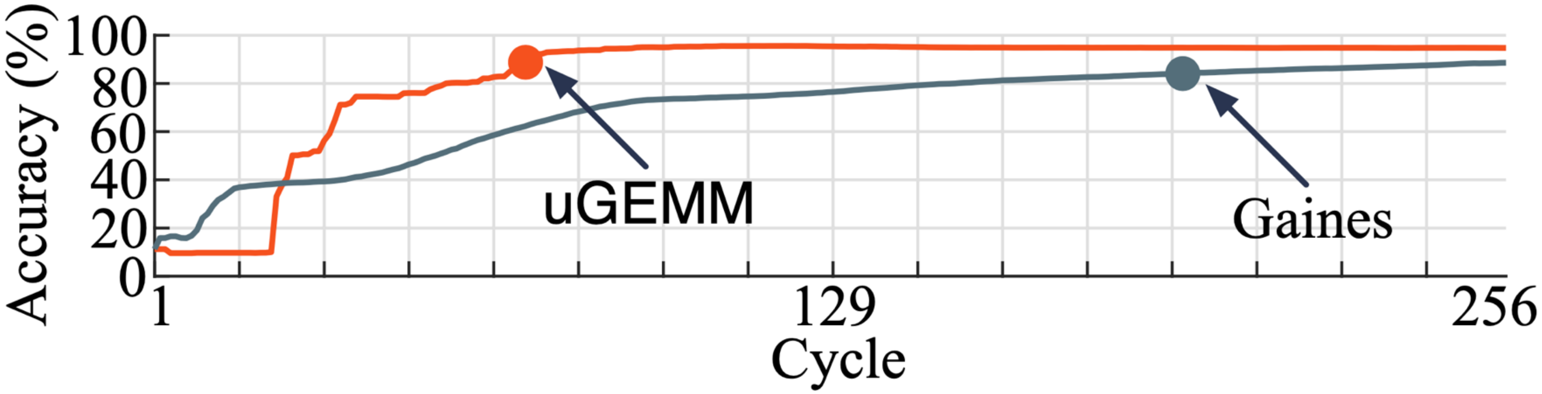
MLP performance

- Final accuracy
  - FP-32bit : 96.87%
  - FXP-8bit: 96.08%
  - uGEMM: 94.7%
  - Gaines: 88.58%
- Cycle count for 95% final accuracy
  - uGEMM: 71
  - Gaines: 195



uGEMM Energy efficiency

- Compared to other unary designs
  - Higher energy efficiency, though slightly higher area
- Compared to binary bit-parallel designs
  - Comparable for matrix multiplication
  - 10X higher for matrix convolution
- Compared to binary bit-serial designs
  - 5X higher for matrix convolution

Thank you!
Q & A